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Finite element exterior calculus – a link between algebraic topology and numerical analysis

(joint work with Douglas N. Arnold, University of Minnesota, and Richard S. Falk, Rutgers University)

Abstract: Scientific computing is today an indispensible tool in most branches of science and engineering. Furthermore, the modeling of increasingly complex phenomena continuously leads to a need for improved understanding of the numerical algorithms. As a consequence, there is an increasing demand for mathematical analysis of computational processes.

The finite element method is one of the greatest advances in numerical computing of the last century. It has become a key tool for simulations of a wide varity of phenomena modeled by partial differential equations. A tremendeous asset of finite elements is that they not only provide a methodology to construct numerical algorithms, but also a theoretical framework for analyzing the algorithms. Still, the construction of accurate finite element methods for systems of differential equations is a rather subtle matter. A necessary requirement for constructing converging numerical schemes is numerical stability, i.e., the discrete equations should be well–posed uniformly in the discretization parameters. For many important problems, the development of stable finite element methods remains extremely challenging, or even out of reach.

Finite element exterior calculus is an approach to the design of stable finite element discretizations for a wide variety of systems of partial differential equations. Stability is achieved by developing discretizations which are compatible with the geometric and algebraic structures, such as de Rham cohomology and Hodge decompositions, which underlie well–posedness of the system of partial differential equations being solved. Instead of considering the design of discrete approximations for each particular problem separately, it has proved beneficial to simultaneously study approximations of an entire differential complex. In this talk we will give an overview of finite element exterior calculus, and explain how this approach has led to improved algorithms for several important problems.