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Locality and unitarity in the structure of quantum cellular automata

Abstract: A quantum cellular automaton is a discrete time quantum lattice system with strictly finite propagation speed. As in the classical case, the update of each cell depends on its neighbours, which means that each cell typically contributes information to several neighbours. Since copying information is forbidden in the quantum case, this implies constraints on the update rules, resulting in a non-trivial interplay between the requirements of unitarity and locality (finite propagation neighbourhood). Cellular automata avoiding the cloning problem can be easily be written down by specifying a local mechanism, like a gate array. We show that such a local mechanism exists for every quantum cellular automaton (in any dimension), provided we may use some additional local work space (local ancillas). When this is not allowed, we show that even in one lattice dimension not every automaton can be implemented. The problem is decided by determining all locally computable properties of cellular automata, which are summarized in a single rational number, the index of the autmaton. The index turns out to label the connected components of the group of cellular automata. Moreover, an automaton can be implemented locally without ancillas iff its index is trivial.