

**Richard Taylor** (Harvard University, Cambridge MA, USA)  
*The Sato-Tate conjecture*

**Abstract:** Much attention has been paid to the counting of the number of solutions to polynomial congruences, i.e. the number of solutions to the polynomial equations in the integers modulo  $m$  for some  $m$ . Some of the hardest questions involve fixing the equation(s) and letting the modulus  $m$  vary over prime numbers. In this case one can ask for an algorithm which given the modulus  $m$  predicts the number of solutions (reciprocity laws) or about the distribution of the number of solutions as  $m$  varies (density theorems). The two sorts of questions are not surprisingly related. Reciprocity laws go back to Gauss' celebrated law of quadratic reciprocity and density theorems to Dirichlet's theorem on primes in an arithmetic progression. Until recently almost all the results known concerned a single polynomial in one variable. Following Wiles' breakthrough in the early 90's, much progress has been made on more complicated reciprocity laws like the Shimura-Taniyama conjecture on elliptic curves (a single cubic equation in two variables). In this talk I will discuss the Sato-Tate conjecture, which has recently been proved in many cases. This is a density theorem for elliptic curves. Interestingly its proof involves proving reciprocity theorems for equations in arbitrarily many variables. I will describe the conjecture and try to place it in a more general context. I will then indicate some of the ideas involved in its proof.