

H. Mete Soner (College of Administrative Sciences and Economics, Koc University, Istanbul, Turkey)

Nonlinear parabolic PDEs and pricing intervals

Abstract: In complete financial markets, the price of any financial instrument is determined uniquely. Further, any derivative instrument can be perfectly replicated by the underlying stocks. However, in the presence of market friction such transactions costs or constraints, such replication is no longer possible. Moreover, there is no unique price. In such markets, one has to give up the idea of replication and may consider investments that super-replicate, i.e., investments whose random future value is almost surely non-negative. Then, the minimal initial wealth of such portfolios form an upper bound for all possible prices. Analogously, one considers the dual problem to obtain the lower bound.

The problem of characterizing the minimal super-replication cost is a non-standard optimal control problem. Using the newly developed dynamic programming for this class of problems [2], one derives the corresponding dynamic programming equation. Conversely, for any nonlinear parabolic PDEs, a corresponding "pricing" problem exists. Thus providing a stochastic representation for these nonlinear equations [1] and also makes it possible for novel Monte-Carlo type simulations.

REFERENCES

- [1] Cheridito, P., Soner, H.M., Touzi, N., and Victoir, N. (2007). Second order backward stochastic differential equations and fully non-linear parabolic PDEs, *Communications on Pure and Applied Mathematics*, 60 (7): 1081-1110.
- [2] Soner H.M., and Touzi N. (2002). Dynamic programming for stochastic target problems and geometric flows, *J. European Math. Soc.*, 4, 201–236.