Laure Saint-Raymond **Monday, July 14, 13:00–13:45, Room C+D**
(Laboratoire Jacques-Louis Lions, Université Pierre et Marie Curie, Paris, France)
Some results about the sixth problem of Hilbert

Abstract: The aim of this lecture is to present some mathematical results describing the transition from kinetic theory, and more precisely from the Boltzmann equation for perfect gases to hydrodynamics. Different fluid asymptotics will be investigated, starting always from solutions of the Boltzmann equation which are only assumed to satisfy the estimates coming from physics, namely some bounds on mass, energy and entropy.

We will introduce some tools for the derivation of these hydrodynamic limits. We will first comment on the entropy inequality which provides uniform a priori estimates on the distribution (depending on the scaling to be considered). We will then explain how these bounds, especially that on the entropy dissipation, allow to control the relaxation mechanism towards local thermodynamic equilibrium. We will finally study the balance between that relaxation process due to collisions, and the other important physical mechanism, namely the free transport : in viscous regimes the global structure of the scaled Boltzmann equation is actually of hypoelliptic type, and one can exhibit some regularizing effect of the free transport.

The incompressible Navier-Stokes limit is the only hydrodynamic asymptotics for which we are able to implement all these tools and for which an optimal convergence result is known. By "optimal", we mean that this convergence result holds globally in time, does not require any assumption neither on the initial velocity profile nor on the initial thermodynamic fields, and that it takes into account boundary conditions, and describes their limiting form.

The state of the art about the incompressible Euler limit is not so complete. Due to the lack of regularity estimates in inviscid regimes, the convergence results describing the incompressible Euler asymptotics of the Boltzmann equation require in particular some additional regularity assumptions on the solution to the target equations.