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Equivariant non-commutative topology

Abstract: Equivariant non-commutative topology is the study of topological invariants of operator algebras—also called C^*-algebras—equipped with additional symmetry such as an action of a group, groupoid, or quantum group. A topological invariant is a functor with some exactness and homotopy invariance properties that is invariant under Morita equivalence. Since equivariant bivariant K-theory is the universal such invariant, it is the main object of study. I will first recall the basic properties of bivariant K-theory and contrast them with those of the stable homotopy category, which plays a similar universal role in in more classical algebraic topology.

The main focus of this talk is a general programme for studying triangulated categories. This programme is motivated by the Adams spectral sequence, but we will apply it to bivariant K-theory. The starting point is some stable homological functor, which serves as a probe for our category. This generates a homological ideal, which allows to carry over notions from homological algebra like projective resolutions and derived functors (relative to the chosen ideal). These derived functors agree with those in a certain Abelian category attached to the situation. This Abelian approximation can often be described concretely in terms of partially defined adjoint functors. It is related to the original category by a spectral sequence.

Finally, this general programme is applied to some examples from equivariant bivariant K-theory. I plan to discuss the Baum–Connes conjecture for locally compact groups, the first example to be considered, and bundles of C^*-algebras over some finite non-Hausdorff spaces. The latter situation has applications to the C^*-algebra classification programme