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Renormalization, Galois symmetries and motives

Abstract: Work of Connes and Kreimer showed that the procedure of renormalization in perturbative quantum field theory known as the BPHZ method, when formulated using Dimensional Regularization and Minimal Subtraction as an associated regularization procedure, admits a geometric interpretation as the Birkhoff factorization of loops in the Lie group of complex points of the affine group scheme dual to a commutative Hopf algebra of Feynman graphs. Inspired by the work of Connes–Kreimer and by evidence collected in Feyman diagrams calculations by Broadhurst and Kreimer indicating the presence of multiple zeta values, Pierre Cartier conjectured that the Connes–Kreimer Hopf algebra should be related to the symmetries of multiple zeta values, and hence to the motivic Galois groups of mixed Tate motives. One difficulty in approaching this conjecture of Cartier was the fact that the Connes-Kreimer Hopf algebra is dependent on the particular physical theory one is considering, while the symmetries of multiple zeta values and of mixed Tate motives are not a priori associated to any particular physical theory.

In joint work with Connes we proved this conjecture of Cartier. The main step consisted of showing that the divergences of a renormalizable QFT can be parameterized by a class of differential systems with irregular singularities (flat equisingular connections) on a principal G bundle (G being the group of characters of the Connes-Kreimer Hopf algebra of a given physical theory) over a space that is the product of an infinitesimal punctured disk (the complexified dimension z of DimReg) and a non-zero complex parameter μ^z, which fixes the energy scale. This formulation of the data of perturbative renormalization makes it possible to unify the data of different theories inside a single category of differential systems up to gauge equivalence, the category of flat equisingular vector bundles. We proved that this is a Tannakian category, hence equivalent to the category of finite dimensional linear representations of an affine group scheme. The latter is a semidirect product of the multiplicative group by the pro-unipotent affine group scheme of the free graded Lie algebra with one generator in each degree. By a result of Deligne and Goncharov, this is known to be a motivic Galois group of a category of mixed Tate motives, although we do not have a canonical identification between equisingular connections and Tate motives.