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Very large graphs

Abstract: In many areas of science huge networks (graphs) are central objects of study: the internet, the brain, various social networks, VLSI, statistical physics. To study these graphs, new paradigms are needed: What are meaningful questions to ask? When are two huge graphs "similar"? How to scale down these graphs without changing their fundamental structure? How to generate random examples with the desired properties?

A reasonably complete answer can be given in the case when the huge graphs are dense; in the more difficult case of sparse graphs there are only partial results. One of the main tools is a "limit object" that can be defined for certain growing graph sequences in the form of a 2-variable measurable function. Analytic properties of this function reflect important graph-theoretic properties of the members of the sequence. For example, Szemeredi's Regularity Lemma is equivalent to a compactness result in this setting. Furthermore, a distance of two graphs can be defined (called the cut distance) such that 2-variable functions form precisely the completion of the set of finite graphs in this metric.

This analytic approach allows us to characterize graph properties that are testable, and to exactly formulate, and obtain first results about, the question of characterizing extremal graphs in extremal graph theory.

The talk will survey joint work with Christian Borgs, Jennifer Chayes, Balazs Szegedy, Vera Sos and Katalin Vesztergombi.