Miklós Laczkovich (Department of Analysis, Eötvös Loránd University, Budapest, Hungary)
Whitney constants, twisted sums, and the difference property

Abstract: In the first part we discuss the relations among various constants measuring the stability of affinity and of the Jensen equation in different classes of functions defined on a convex subset of a linear space. In normed linear spaces the constants corresponding to the classes of continuous, bounded or bounded continuous functions are called the Whitney constants of second order. Some of the results: the stability constants of affinity and of the Jensen equation are always of the same order of magnitude; the Whitney constant corresponding to the bounded functions equals the stability constant of the Jensen equation whenever the latter is finite; if D is a bounded convex set with nonempty interior in a normed linear space, then the Whitney constants are of the same order of magnitude on D. The last result is not true for arbitrary convex sets.

The second topic of the talk concerns twisted sums of Banach spaces. This is connected to the the first topic by the following: if every twisted sum of the Banach spaces E and X is trivial and $D \subset X$ is a bounded convex set with nonempty interior, then the stability constants and the Whitney constants on D are finite, and of the same order of magnitude. As an application, we prove a conjecture of F. Cabello Sánchez, J. M. F. Castillo and P. L. Papini: if every δ-Jensen function can be boundedly approximated by a Jensen function on a convex set D, then the stability constants of D are finite. We prove that every twisted sum of E and X is trivial if and only if, whenever $f : X \to E$ is such that $f(x+y) - f(x) - f(y)$ is bounded for $\|x\|, \|y\| < 1$, then $f = g + A$, where g is bounded for $\|x\| < \delta$ and A is additive.

This leads to the third topic of the talk. We say that the class $C(X, E)$ of continuous functions $f : X \to E$ has the difference property (d.p.) if, whenever $f : X \to E$ is such that $f(x + h) - f(x)$ is continuous for every $h \in X$, then $f = g + A$, where g is continuous and A is additive. It is a general open problem to decide whether or not $C(X, E)$ has the d.p. for a given pair of spaces X and E. We show that if every twisted sum of E and X is trivial, then $C(X, E)$ has the d.p. We show that the classes $C(l_1, \mathbb{R})$ and $C(l_p, l_p)$ $(p > 1)$ do not have the difference property.