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Critical phenomena in random matrix theory

Abstract: Local eigenvalue statistics of unitary invariant random Hermitian matrices show remarkable universal behavior as the size of the matrices tends to infinity. The eigenvalue behavior in the bulk of the spectrum is known as GUE statistics, named after the canonical Gaussian Unitary Ensemble. This is described by the so-called sine kernel. The typical local eigenvalue behavior at the edges of the limiting spectrum is described by the Airy kernel, which is closely related to the Tracy-Widom distribution for the largest eigenvalue.

The universal behavior breaks down at special points where one may have a change in the number of intervals in the limiting spectrum as parameters in the random matrix model change. In these critical regimes the local eigenvalue statistics are still classified within certain universality classes, which are now connected with Painlevé transcendents. These are solutions of very special nonlinear differential equations (Painlevé equations) with remarkable properties. They appear in isomonodromy problems for systems of linear ODEs.

In the talk these recent developments will be described as well as their connection to models of non-intersecting random paths. The main technical tool is the Riemann-Hilbert problem for orthogonal polynomials and its asymptotic analysis with the Deift/Zhou method of steepest descent. In the critical regimes a local approximation (parametrix) is constructed with the Riemann-Hilbert problem for the Painlevé equations.