Nikita Karpenko (Université Pierre et Marie Curie, Paris, France)
Essential dimension of finite p-groups
(joint work with A. S. Merkurjev, cf. [3])

Abstract: The notion of the essential dimension $\mathrm{ed}(G)$ of a finite group G over a field F was introduced in [2]. The integer $\mathrm{ed}(G)$ is equal to the smallest number of algebraically independent parameters required to define a Galois G-algebra over any field extension of F. If V is a faithful linear representation of G over F then $\mathrm{ed}(G) \leq \dim(V)$ (cf. [1, Prop. 4.15]). The essential dimension of G can be smaller than $\dim(V)$ for every faithful representation V of G over F. For example, we have $\mathrm{ed}(\mathbb{Z}/3\mathbb{Z}) = 1$ over $\mathbb{Q}$ or any field F of characteristic 3 (cf. [1, Cor. 7.5]) and $\mathrm{ed}(S_3) = 1$ over $\mathbb{C}$ (cf. [2, Th. 6.5]).

The main result of the talk says that if G is a p-group and F is a field of characteristic different from p containing p-th roots of unity, then $\mathrm{ed}(G)$ coincides with the least dimension of a faithful representation of G over F.

References

[1] G. Berhuy and G. Favi, *Essential dimension: a functorial point of view (after A. Merkurjev)*, Doc. Math. **8** (2003), 279–330 (electronic).
[2] J. Buhler and Z. Reichstein, *On the essential dimension of a finite group*, Compositio Math. **106** (1997), no. 2, 159–179.
[3] N. A. Karpenko and A. S. Merkurjev, *Essential dimension of finite p-groups*, Invent. Math., to appear.
[4] Z. Reichstein and B. Youssin, *Essential dimensions of algebraic groups and a resolution theorem for G-varieties*, Canad. J. Math. **52** (2000), no. 5, 1018–1056, With an appendix by János Kollár and Endre Szabó.