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Motivic structures in non-commutative geometry

Abstract: "Non-commutative geometry" is a term with many meanings. I am following the recent usage, which equates non-commutative geometry with "geometry of triangulated categories" – one attempts to treat a triangulated category, or maybe a DG or an A_∞-category, as a geometric object with all that it entails. The desire to do so comes originally from physics, where interesting A_∞-categories such as the Fukaya category appear in the context of topological field theories.

Mathematically, main motivation comes from the usual algebraic geometry, where one attempts to extract information about an algebraic variety X from its derived category $\mathcal{D}^b(X)$ of coherent sheaves. When passing from X to $\mathcal{D}^b(X)$, some information is certainly lost – different varieties can have equivalent derived categories. What is surprising is how much can be recovered. Here are some the invariants of X which are completely determined by $\mathcal{D}^b(X)$ (or rather, its A_∞-version):

1. algebraic K-theory $K^\bullet(X)$,
2. differential forms Ω^i_X – these correspond to Hochschild homology classes of $\mathcal{D}^b(X)$,
3. de Rham cohomology $H^\bullet_{DR}(X)$ – this corresponds to periodic cyclic homology of $\mathcal{D}^b(X)$.

It is expected that much more is true: loosely speaking, all the "motivic" structures which exits on the de Rham cohomology $H^\bullet_{DR}$ should also exist on the periodic cyclic homology of a nice enough A_∞-category $\mathcal{C}$.

In the talk, I will present a recent discovery in this direction: it turns out that for a nice enough A_∞-category $\mathcal{C}$ defined over a finite field k, or over Witt vectors $W(k)$, the periodic cyclic homology $HP_\bullet(\mathcal{C})$ carries a natural action of the Frobenius map, and moreover, has a structure of a "filtered Diedonné module" of Fontaine-Lafaille – the p-analog analog of a mixed Hodge structure. This allows, among other things, to prove a Hodge-to-de Rham degeneration result for $\mathcal{C}$ using the well-known method of Deligne and Illusie. I will also discuss the relation with the notion of syntomic cohomology, and present a p-adic non-commutative version of the Beilinson conjecture and the regulator map.