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The continuous limit of large random planar maps

Abstract: Planar maps are graphs embedded in the two-dimensional sphere $S2$, considered up to continuous deformation. They have been studied extensively in combinatorics, but they also have significant geometrical applications. Random planar maps have been used in theoretical physics, where they serve as models of random geometry. Our goal is to discuss the convergence of rescaled random planar maps viewed as random metric spaces. More precisely, we consider a random planar map $M(n)$, which is uniformly distributed over the set of all planar maps with n vertices in a certain class. We equip the set of vertices of $M(n)$ with the graph distance rescaled by the factor $n^{-1/4}$. We then discuss the convergence in distribution of the resulting random metric spaces as $n \to \infty$, in the sense of the Gromov-Hausdorff distance between compact metric spaces. This problem was stated by Oded Schramm in his 2006 ICM paper, in the special case of triangulations. In the case of bipartite planar maps, we first establish a compactness result showing that a limit exists along a suitable subsequence. We then prove that this limit can be written as a quotient space of the so-called Continuum Random Tree (CRT) for an equivalence relation which has a simple definition in terms of Brownian labels atttached to the vertices of the CRT. This limiting random metric space had been introduced by Marckert and Mokkadem and called the Brownian map. It can be viewed as a "Brownian surface" in the same sense as Brownian motion is the limit of rescaled discrete paths. We show that the Brownian map is almost surely homeomorphic to the sphere $S2$, although it has Hausdorff dimension 4. Furthermore, we are able to give a complete description of the geodesics from a distinguished point (the root) of the Brownian map, and in particular of those points which are connected by more than one geodesic to the root. As a key tool, we use bijections between planar maps and various classes of labeled trees.

References

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