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Homotopy theory and automorphism groups

Abstract: An important invariant of a discrete group G is its group homology $H_*(G)$. This can be defined in various ways, for example as the homology of a space BG which has fundamental group G and contractible universal covering space. The automorphism group $\text{Aut}(F_n)$ of a free group on n letters can be viewed as a non-commutative analogue of the general linear group $GL(n, \mathbb{Z})$. It is an interesting question to calculate the group homology of $\text{Aut}(F_n)$. I will discuss my calculation of $H_*(\text{Aut}(F_n))$ in the limit where $n \rightarrow \infty$.

The answer is most easily explained in terms of symmetric groups. The symmetric group S_n acts on F_n by permuting generators, and gives map $S_n \rightarrow \text{Aut}(F_n)$. I proved that the induced map $H_*(S_n) \rightarrow H_*(\text{Aut}(F_n))$ is an isomorphism in the limit $n \rightarrow \infty$.

The proof is mostly homotopy theoretical. It uses spaces of graphs to construct good models for $B\text{Aut}(F_n)$.