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The solvability of differential equations

Abstract: In the 1950's, the general mathematical opinion was that all non-degenerate linear partial differential equations were solvable. Therefore, it came as a surprise when Hans Lewy in 1957 found a non-solvable complex vector field. The vector field is a natural one, it is the Cauchy-Riemann operator on the boundary of a strictly pseudo-convex domain. It turned out that almost all linear partial differential equations are unsolvable, because of the Hörmander bracket condition.

A rapid development led to the conjecture by Nirenberg and Treves in 1969: that condition (Ψ) is necessary and sufficient for the solvability of differential equations of principal type. Equations of principal type are essentially those which have simple characteristics, and condition (Ψ) only involves the sign changes of the coefficients of the imaginary part of the highest order terms of the equation.

The Nirenberg-Treves conjecture has recently been proved, see Annals of Mathematics, 163:2, 2006. We shall present the background and the main ideas of the proof. We shall also present some examples and generalizations to systems of differential equations.