José A. Carrillo (Universitat Autònoma de Barcelona, Spain)
The Patlak-Keller-Segel model: free energies, geometric inequalities and gradient flows

Abstract: We will analyze long time asymptotics and blow-up results for nonlinear nonlocal diffusion problems arising in the modelling of chemotaxis in cell movement.

We will first discuss the critical case for the parabolic-elliptic Patlak-Keller-Segel (PKS) model in $\mathbb{R}_2$. Under hypotheses of integrable initial data with finite second moment and entropy, local in time existence for any mass of "free-energy solutions", namely weak solutions with some free energy estimates, is obtained and their lifespan depend on the control of the entropy. Global existence of free-energy solutions with initial data as before for the critical mass $8\,\pi/\chi$ is shown. Moreover, we prove that solutions aggregate as a delta Dirac at the center of mass when $t \to \infty$ keeping constant their second moment at any time. Solutions corresponding to infinite initial second moment will be also discussed.

On the other hand, we generalize some of these ideas and strategies to a nonlinear diffusion version in dimension $d \geq 3$ of the PKS model. The non-linear diffusion is chosen in such a way that its scaling and the one of the Poisson term coincide. We exhibit that the qualitative behaviour of solutions is decided by the initial mass of the system. Actually, there is a sharp critical mass M_c such that if $M \in (0, M_c]$ solutions exist globally in time, whereas there are blowing-up solutions otherwise. While characterising the eventual infinite time blowing-up profile for $M = M_c$, we observe that the long time asymptotics might be much more complicated than in the classical Patlak-Keller-Segel system in dimension two.

This is a summary of results in different works in collaboration with A. Blanchet, E. Carlen, P. Laurenot and N. Masmoudi.