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On the discretization of differential forms

Abstract: A large class of physical models, based on Partial Differential Equations (PDEs), presents a relevant mathematical structure which can be conveniently studied by exploiting tools of differential geometry and algebraic topology. This is the case for all those PDEs whose unknowns can be thought as differential forms: examples are diffusion equations, electromagnetics and elasticity models.

When designing computational tools for these problems, it is desirable to have numerical methods able to inherit or mimic fundamental properties of the underlying PDEs such as, e.g., conservations or symmetries. If the unknowns are differential forms, this goal can be achieved by constructing discretizations of spaces of differential forms which are *compatible* with the geometric structure: e.g., forming suitable complexes.

Along the talk, I will present recent results in this research area which goes under the name of *discrete exterior calculus.* Some of the known finite elements, finite differences and finite volumes techniques fall in a framework which allows for a unified analysis and for the development of new schemes [2].

Finally, the use of these techniques on real-life problems opens new challenges such as the definition of high order discretizations and the robust numerical treatment of engineered-type geometries. I will illustrate promising research directions which might provide a breakthrough along the next years.

References

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