Christoph Böhm (Mathematisches Institut, Westfälische Wilhelms-Universität Münster, Germany)
Ricci flow in higher dimensions
(joint work with Burkhard Wilking)

Abstract: In 1982 Hamilton introduced Ricci flow in order to prove that on a closed 3-manifold the (normalized) Ricci flow evolves a Riemannian metric of positive Ricci curvature to a limit metric of constant positive sectional curvature. It follows that a closed 3-manifold admitting a Riemannian metric of positive Ricci curvature must be a spherical space form. For arbitrary initial metrics the Ricci flow may develop singularities. On 3-manifolds these singularities have been classified by Perelman, geometrically and topologically. This allowed Perelman to prove that on closed 3-manifolds there exists a Ricci flow with surgery. As a consequence one obtains a proof of the Poincaré conjecture.

In higher dimensions the (normalized) Ricci flow will also evolve sufficiently positively curved initial metrics to a limit metric of constant positive sectional curvature. On closed manifolds this is for instance true for Riemannian metrics with positive curvature operator. For arbitrary initial metrics the Ricci flow will however develop much more complicated singularities than in dimension three. In order to reduce the possible singularity models one is led to consider only initial metrics which fulfill curvature conditions which are invariant under the Ricci flow. We introduce such curvature conditions and describe properties of the corresponding class of Riemannian manifolds. Also applications to Einstein manifolds will be given.